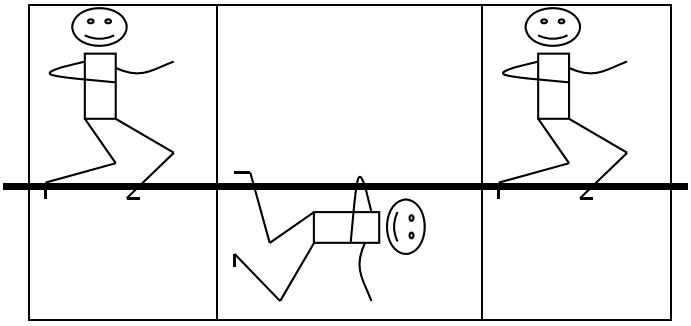
Activity 33 Inverse transformations

Aim: Work with inverse matrices, and explore relationships between the inverse of a linear transformation matrix and the transformation returning an object to its original position.

After each manoeuvre, the acrobat can regain his position by performing the "inverse" manoeuvre. In the case illustrated, a clockwise 90° rotation followed by a counter-clockwise 90° rotation restores him to his original position.



If $\mathbf{T} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then its inverse is $\mathbf{T}^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ since $\mathbf{TT}^{-1} = \mathbf{T}^{-1}\mathbf{T} = \mathbf{I}$.

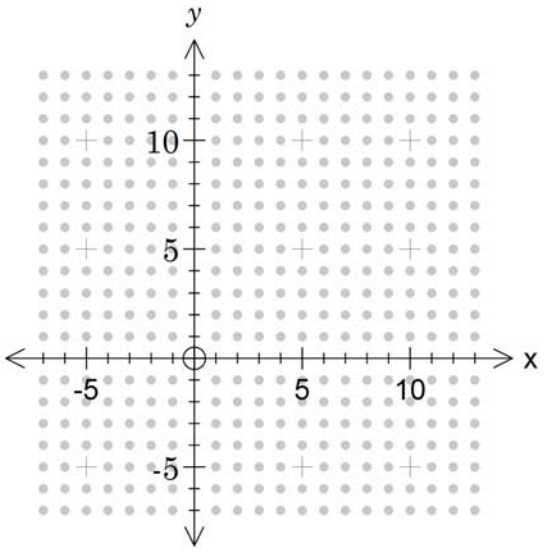
1. Explain why the definition $\mathbf{TT}^{-1} = \mathbf{T}^{-1}\mathbf{T} = \mathbf{I}$ means only square matrices can have inverses.

2. For $\mathbf{T} = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$, $\mathbf{T}^{-1} = \begin{bmatrix} 4 & -1 \\ -1.5 & 0.5 \end{bmatrix}$:

   a) Verify that $\mathbf{TT}^{-1} = \mathbf{T}^{-1}\mathbf{T} = \mathbf{I}$.

   b) Find the image of the rectangle OABC, where O(–1,0), A(–1,1), B(1,1) and C(1,0), under the transformation **T**.

   Sketch the rectangle and its image under **T**.



3. If $\mathbf{T}(\Delta ABC)$ is $\Delta A'B'C'$, where A′(–5, –19), B′(7, 27) and C′(2, 4), find the points A, B and C.

4. Some transformations have no inverse.

   a) Give an example.

   b) Under what conditions does the transformation, $\mathbf{T} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ have no inverse?

Extension

5. Some transformations are their own inverses.

   a) Give three examples of 2×2 matrices that are their own inverses.

   b) Describe the transformations.

   c) Solve $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \mathbf{I}$ to find all transformations that are their own inverses.

Learning notes

Questions 1 and 2 connect the algebraic (matrix) and geometrical views of transformations.

Question 2 can be done neatly using a split screen: Main and Geometry.

Q3 requires use of the inverse transformation matrix.

Q5 is beyond the scope of this course. Set up and solve the four simultaneous equations in four unknowns that are not linear. The solution will still need to be interpreted.